10/5/17

"Everyone has a gift, some people just never open theirs." -Mr. Callahan

HW: "2017 A2 CC Complex Numbers" finish the packet

Test 2 on Monday 10/16

AIM: What are Complex Numbers?

Warm Up:

1. $625^{\frac{3}{4}} = \sqrt[4]{625^3}$

$= \left(\sqrt[4]{625}\right)^3$

2. $(3ab^2c)\left(\frac{2a^2b}{c^3}\right)^{-1}$

$\left(\frac{3ab^2c}{1}\right)\left(\frac{c^3}{2a^2b}\right)^1$

$\frac{3ab^2c^4}{2a^2b} = \boxed{\frac{3bc^4}{2a}}$

3. Solve: $x^2+1=0$

$-1 \quad -1$

$x^2 = -1$

$\sqrt{x^2} = \sqrt{-1}$

$\boxed{x = \sqrt{-1}}$

$\boxed{x = i}$

Definition: The imaginary unit, i, is defined as $\sqrt{-1}$. Therefore:

$i^0 = \left(\sqrt{-1}\right)^0 = \boxed{1}$

$i^1 = \boxed{i}$

$i^2 = \sqrt{-1}\sqrt{-1} = \boxed{-1}$

$i^3 = i \cdot i^2 = i(-1) = \boxed{-i}$

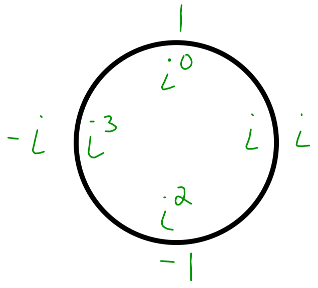
$i^4 = i^2 \cdot i^2 = (-1)(-1) = \boxed{1}$

$i^5 = i \cdot i^4 = i(1) = \boxed{i}$

$i^6 = i^3 \cdot i^3 = (-i)(-i) = i^2 = \boxed{-1}$

$i^7 = i^4 \cdot i^3 = (1)(-i) = \boxed{-i}$

We can easily simplify any power of i. We do this by:



✱ Divide the exponent by 4 and then only count the remainder.

$.25 = i$

$.50 = -1$

$.75 = -i$

No decimal $= 1$

Examples:

Simplify each.

1. $i^{20} = 1$

$20 \div 4 = 5$

2. $i^{39} = \boxed{-i}$

$39 \div 4 = 9.\underline{75}$

3. $i^{78} = \boxed{-1}$

$78 \div 4 = 19.50$

4. $3i^{11} \cdot 2i^5$

$6i^{16}$

$6(1) = \boxed{6}$

$16 \div 4 = 4.$

$i^{16} = 1$

Property of negative square roots:

$$\sqrt{-c} = \sqrt{-1c} = \sqrt{-1}\sqrt{c} = i\sqrt{c}$$

$\sqrt{-1}$ $\sqrt{c}$

$i\sqrt{c}$

* Shortcut (-) under radical take it out as i

Examples:

Simplify each.

5. $\sqrt{-25}$

$i\sqrt{25}$

$i\,5$

$\boxed{5i}$

6. $\sqrt{-32}$

$i\sqrt{32}$

$\sqrt{16}\sqrt{2}$

$i\,4\sqrt{2}$

$\boxed{4i\sqrt{2}}$

7. $-\sqrt{25}-\sqrt{-147}$

$-5 \quad i\sqrt{147}$

$i\sqrt{49}\sqrt{3}$

$\boxed{-5-7i\sqrt{3}}$

8. $\sqrt{-128}$

$i\sqrt{128}$

$\sqrt{64}\sqrt{2}$

$8i\sqrt{2}$

9. $\sqrt{-9}+\sqrt{-16}$

$3i + 4i = \boxed{7i}$

Definition:

A number of the form $a+bi$, where a and b are real numbers and $i=\sqrt{-1}$ is called a **complex number.** a is called the **real part** and bi is called the **imaginary part**. A complex number written with the real part first and the imaginary part last is in **standard form.**

real imaginary

Examples: $(x+2)\cdot(x-3)$ → $a+bi$

Perform the operations and put your answers in standard form.

10. $(-1+2i)+(5-3i)$

$\boxed{4-i}$

11. $(-11-40i)-(2+10i)$

$-11-40i-2-10i$

$\boxed{-13-50i}$

$i^2=-1$

12. $10i(6-8i)$

$60i-80i^2$

$60i-80(-1)$

$60i+80$

$\boxed{80+60i}$

13. $(2+5i)(3-15i)$

$6-30i+15i-75i^2$

$-75(-1)$

$6-15i+75$

$81-15i$

14. $\sqrt{-4}\cdot\sqrt{-10}\cdot\sqrt{36}$

$i\sqrt{4}\cdot i\sqrt{10}\cdot 6$

$i^2\sqrt{40}\cdot 6$

$-1\sqrt{40}\cdot 6$

$-6\sqrt{40}$

$-6\sqrt{4}\sqrt{10}$

$-6\cdot 2\sqrt{10}$

$\boxed{-12\sqrt{10}}$

15. $(5-\sqrt{-27})-(9+\sqrt{-108})$

$5-\sqrt{-27}-9-\sqrt{-108}$

$-4-\sqrt{-27}-\sqrt{-108}$

$\sqrt{-9}\sqrt{3}$ $\sqrt{-36}\sqrt{3}$

$-4-3i\sqrt{3}-6i\sqrt{3}$

$\boxed{-4-9i\sqrt{3}}$

16. $(-2+6i)(3-2i)$

$-6+4i+18i+12\cancel{i^2}$

$-6+22i+12$

$\boxed{6+22i}$

17. $(4+i)(-5-3i)$

$-20-12i-5i-3i^2$

$-20-17i+3\cancel{i^2}$

$-20-7i+3$

$\boxed{-17-7i}$

$18\div4=4.5$ $25\div4=6.25$ $28\div4=7$ $43\div4=10.75$

18. Simplify: $5i^{18}+7i^{25}+2i^{28}+6i^{43}$

$5(-1)+7(i)+2(1)+6(-i)$

$-5+7i+2-6i$

$\boxed{-3+1i}$

19. Determine the result in simplest $a+bi$ form:

$(5+2i)(-3+i)+4i(2+3i)$

$-15+5i-6i+2\cancel{i^2}$ $\quad +8i+12\cancel{i^2}$

$(-17-i)$

$-17-i+8i-12$

$\boxed{-29+7i}$

Definition:

$a+bi$ and $a-bi$ are called **complex conjugates.** So, $(a+bi)(a-bi)=$

$(a+bi)(a-bi)$

$a^2-abi+abi-b^2i^2$

$a^2+b^2\cancel{i^2}$

a^2+b^2

4) a) $\sqrt{x^2} = \sqrt{-60}$

$x = \sqrt{-60}$

$x = \sqrt{-4}\ \sqrt{15}$

$x = 2i\sqrt{15}$

3) a) $(5+2i)+(3-2i)$

$5+2i+3-2i$

$\boxed{8+0i}$

$\underline{a+bi}$